Lunar system constraints on the modified theories of gravity

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The Modified gravity's approach to the missing mass problem in galaxies requires introducing a functional that is to be identified through observations and experiments.

We report that the accurate value of the Earth GM measured by the Lunar Laser Ranging and that by various artificial Earth satellites, including the accurate tracking of the LAGEOS satellites, firmly constraint this functional such that most of the so-far chosen/proposed functional are refuted. This implies that the transition from the Newtonian regime to the MONDian one is to be steeper than what commonly thought.

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The missing mass problem in galaxies can be resolved either by the Modified Newtonian Dynamics (MOND), or the Modified Gravity (MOG) or the dark matter hypothesis. The first two paradigms require introducing a functional that interpolates the Newtonian regime to the MOND regime. This functional is to be identified by examining the data.

MOND [1] alters the Newton's second law of dynamics to

$$F = mf(\frac{|a|}{a_0})\vec{a} \tag{1}$$

wherein F is the total force exerted on the particle and [2]

$$a_0 = 1.0 \times 10^{-10}\frac{m}{s^2}\,. \tag{2}$$

and $f(x)$ is a function possessing the following asymptotical behaviors

$$f(x) = \begin{cases} 1 \ , \ x \gg 1 \\ x \ , \ x \leq 1 \end{cases} \tag{3}$$

MOND is to be applied in a frame inertial with respect to the frame wherein the CMB background is isotropic. The second law of Newtonian dynamics in the presence of the earth gravitational acceleration has been tested in lab up to the acceleration of $10^{-11}\frac{m}{s^2}$ [5], and $10^{-14}\frac{m}{s^2}$ [5]. No deviation has been observed. Nonetheless these experiments are not performed in a frame inertial with respect to the CMB frame. They are perfumed in an accelerating frame with respect to the CMB frame: the earth. So they imply no conclusive result on the MOND.

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The second approach is theories of modified gravity. We consider AQUAL (aquadratic Lagrangian theory) model [3]. AQUAL alters the Newtonian gravitational potential equation

$$\nabla^i \nabla_i \Phi_N = 4\pi G\rho, \tag{4}$$

to

$$\nabla^i \left(\mu(\frac{|\nabla \Phi|}{a_0}) \ \nabla_i \Phi \right) = 4\pi G\rho, \tag{5}$$

where μ is a functional in term of the gravitational potential and a_0 is given in (2). The asymptotic behavior of μ coincides to that of f:

$$\mu(x) = \begin{cases} 1 \ , \ x \gg 1 \\ x \ , \ x \leq 1 \end{cases} \tag{6}$$

The AQUAL model fixes only the asymptotic behavior of the functional μ. Various functionals possessing these asymptotic behaviors have been suggested for μ, including :

$$\text{Ref. [6]} \ : \ \mu_1(x) = \frac{x}{1+x} \tag{7a}$$

$$\text{Ref. [7]} \ : \ \mu_2(x) = \frac{\sqrt{1+4x}-1}{\sqrt{1+4x}+1} \tag{7b}$$

$$\text{Ref. [1]} \ : \ \mu_3(x) = \frac{x}{\sqrt{1+x^2}} \tag{7c}$$

$$\text{Ref. [8]} \ : \ \mu_4(x) = \frac{6x}{\pi^2} \int_0^{\frac{\pi^2}{6x}} dz \frac{z}{e^z - 1} \tag{7d}$$

$$\text{Ref. [9]} \ : \ \mu_5(x) = x\frac{x^3-1}{x^4-1} \tag{7e}$$

In this paper, we study the earth Lunar system constraints on the above functionals. In so doing we note that the accurate value of the mass ratio of the Sun/(Earth+Moon) from the Lunar Laser Ranging combined with the Solar GM and the lunar GM from lunar orbiting spacecrafts [10] yields the effective gravitational mass of the Earth in an Earth-centered reference frame with the precision of one part in 10^8 :

$$GM^{LLR}_{Earth}(r_{LD}) = 398600.443 \pm 0.004 \frac{km^3}{s^2} \tag{8}$$

where r_{LD} represents the Lunar distance: the average distance between the Earth and Moon. The effective gravitational mass is defined to be the gravitational field multiplied by r^2 where r is distance from the center of the earth. The effective gravitational mass of the Earth has also been measured by various artificial Earth satellites , including the accurate tracking of the LAGEOS

satellites orbiting the Earth in nearly circular orbits with semimajor axes about twice the radius of the Earth [11]:

$$GM_{Earth}^{LAGEOS}(2r_{Earth}) = 398600.4419 \pm 0.0002\frac{km^3}{s^2} \tag{9}$$

where r_{Earth} stands for the radius of the Earth.

In order to show how (8) and (9) constraint the AQUAL model we first need to compute the gravitational field of the Earth-Moon system in the AQUAL model. In so doing note that the general solution to the AQUAL model can be expressed in term of the corresponding Newtonian gravitational potential

$$\mu(\frac{|\nabla\Phi|}{a_0})\nabla\Phi \; = \; \nabla\Phi_N + \nabla\times\vec{h} \tag{10}$$

where $\vec{h}$ is identified by

$$0 \; = \; \nabla\times\nabla\Phi \; = \; \nabla\times(\frac{\nabla\Phi_N + \nabla\times\vec{h}}{\mu(\frac{|\nabla\Phi|}{a_0})}) \tag{11}$$

In the two-body approximation to the Earth-Moon system, considering the fact that the mass of Moon is much smaller than the Earth's mass, and higher gravitational moments can be neglected, the Newtonian gravitational field around the earth is spherical. This in turn implies that $\vec{h}$ is vanishing for the Earth-Moon system. So the gravitational acceleration near the earth, in the AQUAL model holds

$$\mu(\frac{|\nabla\Phi|}{a_0})\nabla\Phi \; = \; \nabla\Phi_N \tag{12}$$

Recalling that near the Earth $\mu \approx 1$, the above equation can be perturbatively solved for $\nabla\Phi$

$$\nabla\Phi \; \approx \; \frac{1}{\mu(\frac{|\nabla\Phi_N|}{a_0})}\nabla\Phi_N \tag{13}$$

Utilizing $\nabla\Phi_N = \hat{r}\frac{GM_{Earth}}{r^2}$ results

$$\nabla\Phi \; \approx \; \frac{\hat{r}}{\mu(\frac{GM_{Earth}}{a_0 r^2})}\frac{GM_{Earth}}{r^2} \tag{14}$$

Now note that (8) represents $r^2\nabla\Phi$ at the Lunar distance:

$$\frac{GM_E}{\mu(\frac{GM_E}{a_0 r_{LD}^2})} = 398600.443 \pm 0.004\frac{km^3}{s^2} \tag{15}$$

while (9) represents $r^2\nabla\Phi$ at $r = 2r_E$:

$$\frac{GM_E}{\mu(\frac{GM_E}{4a_0 r_E^2})} = 398600.4419 \pm 0.0002\frac{km^3}{s^2} \tag{16}$$

Consistency between (15) and (16) then demands that

$$|\frac{1}{\mu(x_{LD})} - \frac{1}{\mu(x_{2r_E})}| < 10^{-8} \tag{17}$$

where

$$x_{LD} = \frac{GM_E}{a_0 r_{LD}^2} = 2.7 \times 10^7 \tag{18}$$

$$x_{2r_E} = \frac{GM_E}{4a_0 r_E^2} = 2.4 \times 10^{10} \tag{19}$$

wherein the lunar distance (LD) is set to 384,400 kilometers and the radius of the earth is taken to be 6,371 kilometers.

Eq. (17) is the Lunar system constraints on the AQUAL functional. Let us calculate $|\frac{1}{\mu(x_{LD})} - \frac{1}{\mu(x_{2r_E})}|$ for the functional chosen by the literature (7):

$$|\frac{1}{\mu_1(x_{LD})} - \frac{1}{\mu_1(x_{2r_E})}| = 3.7 \times 10^{-8} \tag{20}$$

$$|\frac{1}{\mu_2(x_{LD})} - \frac{1}{\mu_2(x_{2r_E})}| = 1.9 \times 10^{-4} \tag{21}$$

$$|\frac{1}{\mu_3(x_{LD})} - \frac{1}{\mu_3(x_{2r_E})}| = 4.4 \times 10^{-16} \tag{22}$$

$$|\frac{1}{\mu_4(x_{LD})} - \frac{1}{\mu_4(x_{2r_E})}| = 1.5 \times 10^{-8} \tag{23}$$

$$|\frac{1}{\mu_5(x_{LD})} - \frac{1}{\mu_5(x_{2r_E})}| = 5.9 \times 10^{-23} \tag{24}$$

This means that the choice of ref. [6] and [8] are refuted by the Lunar system respectively at the confidence level of 3.7 and 1.5 sigma. Note that in particular discarding the choice of [6] means that the AQUAL model does not exhibit the Rindler acceleration. The choice of ref. [7] is refuted by the Lunar constraint at the confidence level of more than 5,000 sigma. Note that this choice is also reported to be refuted in the earth lab [12], and also by the astronomical observations [6]. The lab report of ref. [12] is not sufficient to achieve conclusive result on the functional because it is done in the presence of the gravitational field of the earth, while the Lunar constraint returns conclusive result. Finally note that the choices of ref. [1] and ref. [9] are consistent with the Lunar system. This means that the transition from the Newtonian Regime to the MONDian regime of the AQUAL model is to be steeper than that commonly thought.

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